Finite Temperature Depinning of a Flux Line from a Nonuniform Columnar Defect

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A flux line in a Type-II superconductor with a single nonuniform columnar defect is studied by a perturbative diagrammatic expansion around an annealed approximation. The system undergoes a finite temperature depinning transition for the (rather unphysical) on-the-average repulsive columnar defect, provided that the fluctuations along the axis are sufficiently large to cause some portions of the column to become attractive. The perturbative expansion is convergent throughout the weak pinning regime and becomes exact as the depinning transition is approached, providing an exact determination of the depinning temperature and the divergence of the localization length.

The finite-temperature statistical behavior of magnetic flux lines (FLs) in Type-II superconductors with various kinds of disorder has been a topic of considerable interest and technological relevance following the advent of high-T_c superconductors about a decade ago [1]. Early on, it was realized that compared to point defects, extended defects such as twin boundaries and columnar amorphous regions created by heavy ion irradiation [2] are much more effective in pinning of FLs at higher temperatures, an essential feature for technological applications of these new materials. Most analyses of extended defects studied uniform structures [3]. However, it is natural to expect that such defects contain spatial inhomogeneities in themselves. This paper addresses the implications of such a possibility for the simple case of a single FL in the presence of a single inhomogenous columnar defect with short-range correlated disorder along its axis, which is aligned with the external magnetic field.

A number of closely related works on 2D wetting with quenched disorder [4,5], directed polymers with random interaction [7], strongly inhomogeneous surface growth [6], and 2D wetting and directed polymers in a periodic potential [8,9] have mostly considered a (1+1) dimensional geometry, for which the disorder was marginally relevant at the depinning transition. Higher dimensional systems were glossed over upon the observation that disorder was irrelevant, i. e., the critical behavior near the depinning transition was the same as the uniform defect case. Nevertheless, it would be very useful to compute the change in the free energy and the localization length due to quenched disorder, in some vicinity of the depinning transition where small-scale details of the disorder becomes unimportant. This paper presents a controlled perturbation expansion, which becomes exact in the weak pinning limit, for the corrections to the free energy of the FL due to fluctuations in the pinning potential along the columnar defect.

Consider a single FL (with line tension $\tilde{\epsilon}$) in the presence of a columnar defect of radius b, oriented in the $z-$direction with a pinning potential $U(z)$ inside the defect. The Hamiltonian for a FL configuration $\{\mathbf{r}(z)\}$ is

$$\mathcal{H}(\{\mathbf{r}(z)\}) = \int_0^L dz \left\{ \frac{\tilde{\epsilon}}{2}\left(\frac{d\mathbf{r}}{dz}\right)^2 + [U_0 + \delta U(z)]\, h(\mathbf{r}(z)) \right\}, \tag{1}$$

where $h(\mathbf{r}) = \Theta(b - |\mathbf{r}|)$.($\Theta$ is the unit step function.) The fluctuations $\delta U(z)$ around the mean potential U_0 have a Gaussian distribution with zero mean and correlations

$$\overline{\delta U(z)\delta U(z')} = \frac{\sigma^2}{2\pi}\exp\left(-\frac{(z-z')^2}{2\xi^2}\right), \tag{2}$$

where the overline represents averaging over the disorder potential. Various thermodynamic properties of the system such as the free energy can be deduced from the partition function of the system:

$$Z = Z_{\text{free}}^{-1}\int \mathcal{D}\mathbf{r}(z)\exp\left[-\beta\mathcal{H}(\{\mathbf{r}(z)\})\right], \tag{3}$$

where $\beta = 1/T$ ($k_\mathrm{B} \equiv 1$ throughout), and the normalization factor is chosen such that $Z = 1$ for $U_0 = \delta U(z) = 0$. Z is a random variable that depends on the particular realization of disorder. Thermodynamic quantities typically involve averaging of $\ln Z$, which is usually too difficult to obtain directly. An alternative approach is to calculate moments of the random variable Z since $\overline{Z^n} = \overline{e^{n\ln Z}}$ is the characteristic function for the random variable $\ln Z$ [10]. Indeed,

$$\overline{Z^n} = \exp\left[\sum_{k=1}^{\infty}\frac{n^k}{k!}C_k(\ln Z)\right], \tag{4}$$

where $C_k(\ln Z)$ is the kth cumulant of $\ln Z$. The nth moment of Z is

$$Z^n = \int \mathcal{D}\mathbf{r}_1(z)...\mathcal{D}\mathbf{r}_n(z)\exp\left\{-\beta\int dz \sum_{i=1}^{n}\frac{\tilde{\epsilon}}{2}\left(\frac{d\mathbf{r}_i}{dz}\right)^2 + [U_0 + \delta U(z)]\sum_{i=1}^{n} h(\mathbf{r}_i(z))\right\}. \tag{5}$$

Upon averaging over disorder, one obtains expectation values of the form

$$\overline{e^{-\beta \int dz\, m(z)\delta U(z)}} = e^{\frac{\beta^2}{2}\int dz dz'\, m(z)m(z')\overline{\delta U(z)\delta U(z')}} \approx \exp\left\{\frac{\beta^2\sigma^2\xi}{2}\int dz\,[m(z)]^2\right\} \quad (6)$$

where $m(z) \equiv \sum_{i=1}^{n} h(\mathbf{r}_i(z))$ is the number of lines that are inside the defect at height z, and the approximation $m(z') \approx m(z)$ leading to the final result is good provided that $\xi < \xi_z \equiv \beta\tilde{\epsilon}b^2$, the diffusion length inside the defect. Using $[m(z)]^2 = m(z) + m(z)[m(z)-1]$, the nth moment can be rewritten as

$$\overline{Z^n} = \int \prod_{i=1}^{n} \{\mathcal{D}\mathbf{r}_i(z) \exp[-\beta\mathcal{H}_0(\{\mathbf{r}_i(z)\})]\} \times \exp\left[\beta^2\sigma^2\xi \int dz \sum_{i<j} h(\mathbf{r}_i(z))h(\mathbf{r}_j(z))\right], \quad (7)$$

where

$$\mathcal{H}_0(\{\mathbf{r}_i(z)\}) = \int_0^L dz \left\{\frac{\tilde{\epsilon}}{2}\left(\frac{d\mathbf{r}}{dz}\right)^2 + U_{\text{eff}} h(\mathbf{r}(z))\right\} \quad (8)$$

is the Hamiltonian corresponding to a FL in the presence of a *uniform* columnar defect, with defect energy per unit length

$$U_{\text{eff}}(T) = U_0 - \sigma^2\xi/2T. \quad (9)$$

The solution to the uniform cylindrical defect problem is well studied [3]. The free energy per unit length f_0 and probability density $\psi_0^2(\mathbf{r})$ of the FL at a position $\mathbf{r}$ are given respectively by the ground state energy and wavefunction of the corresponding two-dimensional "Schrödinger Equation"

$$\left[-\frac{T^2}{2\tilde{\epsilon}}\nabla_\perp^2 + U_{\text{eff}} h(\mathbf{r})\right]\psi_0(\mathbf{r}) = f_0\psi_0(\mathbf{r}). \quad (10)$$

For $U_{\text{eff}} < 0$, the FL is pinned to the defect, with

$$f_0 \approx -\frac{|U_{\text{eff}}|}{2}\left(\frac{T}{T_\ell}\right)^2 e^{-2(T/T_\ell)^2},\ T \gg T_\ell, \quad (11)$$

$$\ell_\perp \equiv \left[\int d\mathbf{r}\, r^2\psi_0^2(\mathbf{r})\right]^{1/2} \approx b e^{(T/T_\ell)^2},\ T \gg T_\ell, \quad (12)$$

where $\ell_\perp$ is the localization length and $T_\ell \equiv \sqrt{-U_{\text{eff}}\tilde{\epsilon}}\, b$ is the localization temperature, below which the FL is strongly pinned to the defect ($\ell_\perp \approx b$) and the free energy is dominated by the ground state energy rather than the entropy ($f_0 \approx U_{\text{eff}}$).

For $U_0 < 0$ (on-average attractive defect), $U_{\text{eff}} < 0$ and the FL is pinned to the defect at all temperatures. However, for $U_0 > 0$ (on-average repulsive defect), there is a depinning transition when $U_{\text{eff}} = 0$, at a temperature

$$T_d = \frac{\sigma^2\xi}{2U_0}. \quad (13)$$

For $T > T_d$, the FL is delocalized, with $f_0 = 0$. As $t \equiv (T_d - T)/T_d \searrow 0^+$, the free energy and localization length diverge as

$$f_0(t) \approx -\frac{U_0}{2}\frac{\sigma^4}{t\sigma_c^4}\exp\left[-\frac{2}{t}\frac{\sigma^4}{\sigma_c^4}\right], \quad (14)$$

$$\ell_\perp(t) \approx b\exp\left[\frac{1}{t}\frac{\sigma^4}{\sigma_c^4}\right], \quad (15)$$

where $\sigma_c \equiv U_0^{3/4}\tilde{\epsilon}^{1/4}(2b/\xi)^{1/2}$.

Notice that

$$\overline{Z^n} = Z_0^n \left\langle \exp\left[\beta^2\sigma^2\xi \int dz \sum_{i<j} h(\mathbf{r}_i(z))h(\mathbf{r}_j(z))\right]\right\rangle_0, \quad (16)$$

where $Z_0 = e^{-\beta f_0 L}$ and $\langle ... \rangle_0$ denotes an average over a canonical ensemble of n *non-interacting* FLs in the presence a uniform defect of strength U_{eff}. This additional exponential factor is due to the fact that all n FLs sample the same "quenched" realization of disorder, and can be interpreted as an effective two-body interaction between FLs that is nonzero *only when both FLs are inside the defect.*

For $T > T_b$, the FLs are delocalized and the free energy for the quenched system is identically 0. In the weak pinning regime $T_d > T \gg T_\ell$, corrections to the free energy can be computed as a controlled perturbation series in some "small" parameter. Expanding Eq.(16) in a power series:

$$\frac{\overline{Z^n}}{Z_0^n} = \sum_{k=0}^{\infty}\frac{(\beta^2\sigma^2\xi)^k}{k!}\sum_{i_1<j_1}\cdots\sum_{i_k<j_k}\int dz_1 ... dz_k \times \langle h(\mathbf{r}_{i_1}(z_1))h(\mathbf{r}_{j_1}(z_1))...h(\mathbf{r}_{i_k}(z_k))h(\mathbf{r}_{j_k}(z_k))\rangle_0. \quad (17)$$

The expectation value factors for distinct FLs since each FL wanders independently of each other. After ordering in z such that $z_i \le z_j$ for $i < j$, the product can be further factorized as

$$\left\langle \prod_{i=1}^{s} h(\mathbf{r}(z_i))\right\rangle_0 = \Psi_0^{2s}\prod_{i=1}^{s-1}\frac{\langle h(\mathbf{r}(z_i))h(\mathbf{r}(z_{i+1}))\rangle_0}{\Psi_0^4}, \quad (18)$$

$$\Psi_0^2 \equiv \langle h(\mathbf{r}(z))\rangle_0 = \int_{|\mathbf{r}|<b} d\mathbf{r}\,\psi_0^2(\mathbf{r}) \approx \pi\left(\frac{T}{T_\ell}\right)^4 e^{-2(T/T_\ell)^2}, \quad (19)$$

since each visit to the defect (at $z = z_i$) is a renewal event, i. e., the configuration of the FL for $z > z_i$ is independent of its configuration for $z < z_i$. However, the probability of a subsequent visit to the defect is enhanced up to a distance of order $\ell_z = \xi_z(\ell_\perp/b)^2$, after which the FL "forgets" its previous visit. This enhancement can be captured in an effective "propagator"

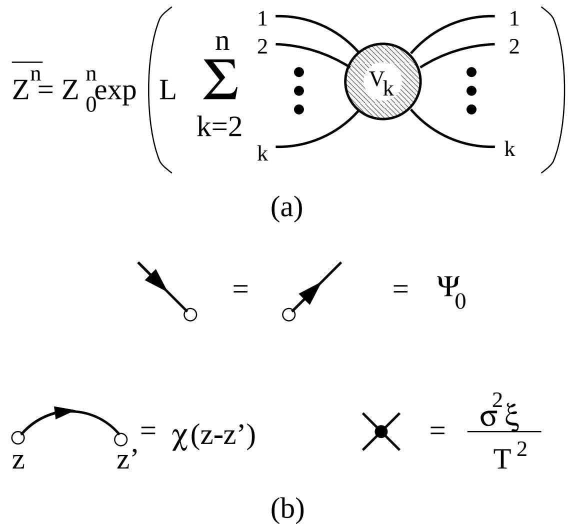

FIG. 1. (a) Diagrammatic representation of the disorder-averaged partition function. (b) Feynman rules for calculating various diagrams that contribute to vertex functions V_k.

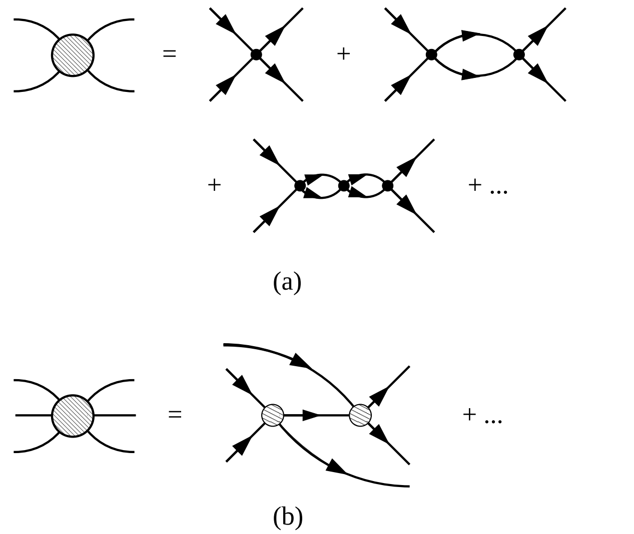

FIG. 2. (a) Diagrammatic representation of the geometric series that contributes to V_2. (b) Leading-order contribution to V_3.

$$\chi(z) \equiv \frac{\langle h(\mathbf{r}(0))h(\mathbf{r}(z))\rangle_0 - \langle h(\mathbf{r}(0))\rangle_0^2}{\langle h(\mathbf{r}(0))\rangle_0^2}, \tag{20}$$

which contains all the $z-$dependence and becomes exponentially small for $z > \ell_z$. Then,

$$\left\langle \prod_{i=1}^{s} h(\mathbf{r}(z_i)) \right\rangle_0 = \Psi_0^{2s} \prod_{i=1}^{s-1} [1 + \chi(z_{i+1} - z_i)]. \tag{21}$$

Upon further manipulation, the disorder-averaged partition function can be recast into a form [See Fig. 1(a)]

$$\overline{Z^n} = e^{-nf_0 L/T} \exp\left[\frac{n(n-1)}{2} L \sum_{k=2}^{\infty} V_k\right], \tag{22}$$

where the vertex functions V_k are a sum over all connected graphs with k incoming and k outgoing lines, constructed from the building blocks shown in Fig. 1(b). The combinatorial factor coming from the choice of the first pair to be contracted is pulled out for notational convenience.

Leading-order contributions to the free energy come from V_2, which can be computed exactly in terms of Ψ_0 and χ: As shown in Fig. 2(a), the terms from a geometric series, and

$$V_2 = \frac{\sigma^2 \xi}{T^2} \Psi_0^4 \sum_{i=0}^{\infty} \left[\frac{\sigma^2 \xi}{T^2} \Psi_0^2 \int_0^{\infty} dz\, \chi^2(z)\right]^i. \tag{23}$$

Just below the pinning temperature, $0 < t \ll 1$, $\Psi_0^2 \int_0^{\infty} dz \chi^2(z) \approx C\xi_z$, where C is a constant of $\mathcal{O}(1)$. In this weak pinning limit, the correction to the disorder-averaged free energy and its variance are given by the coefficients of terms in the exponent of $\overline{Z^n}$ that are linear and quadratic in n, respectively.

$$\overline{\delta f_{\text{quenched}}} \approx \frac{\sigma^2 \xi}{2T_d} \frac{\Psi_0^4}{1 - (C\sigma^2 \xi \xi_z / T^2)}$$
$$\approx \pi^2 U_0 \left(\frac{\sigma^4}{\sigma_c^4 t}\right)^4 \frac{\sigma^4}{\sigma^4 - 2C\sigma_c^4} \exp\left[-\frac{4}{t}\frac{\sigma^4}{\sigma_c^4}\right], \tag{24}$$
$$\overline{\delta f^2_{\text{quenched}}} = \frac{T_d}{L} \overline{\delta f_{\text{quenched}}}. \tag{25}$$

Thus, the free energy is self averaging as $L \to \infty$. Note that the geometric series converges when $\sigma^2 > \sqrt{2C}\sigma_c^2$. The leading-order contribution to V_3 is [See Fig. 2(b)]

$$V_3^{(1)} = (n-1) V_2^2 \int_0^{\infty} dz\, \chi(z). \tag{26}$$

Near the transition, $\int_0^{\infty} dz\, \chi(z) \approx (T/T_\ell)^4 / \Psi_0^2$ and therefore this next correction to $\delta f_{\text{quenched}}$ is smaller by a factor of order $(\sigma^4/\sigma_c^4 t)^4 \exp(-2\sigma^4/t\sigma_c^4)$, in fact, leading order contributions from higher order vertex functions form a series where each term is smaller from the previous one by this same factor. Thus, the asymptotic behavior of the free energy and the localization length near the depinning transition is given by Eqs.(14) and (15), since

$$\lim_{T \to T_d^-} \frac{\overline{\delta f_{\text{quenched}}}}{f_0} = 0. \tag{27}$$

The perturbation series can be used to compute corrections to the free energy even away from the depinning transition, provided that $T \gg T_\ell$, i. e., within the weak pinning regime. The situation is clear for $U_0 < 0$. In the interesting case $U_0 > 0$ and $\sigma \gg \sigma_c$, this condition

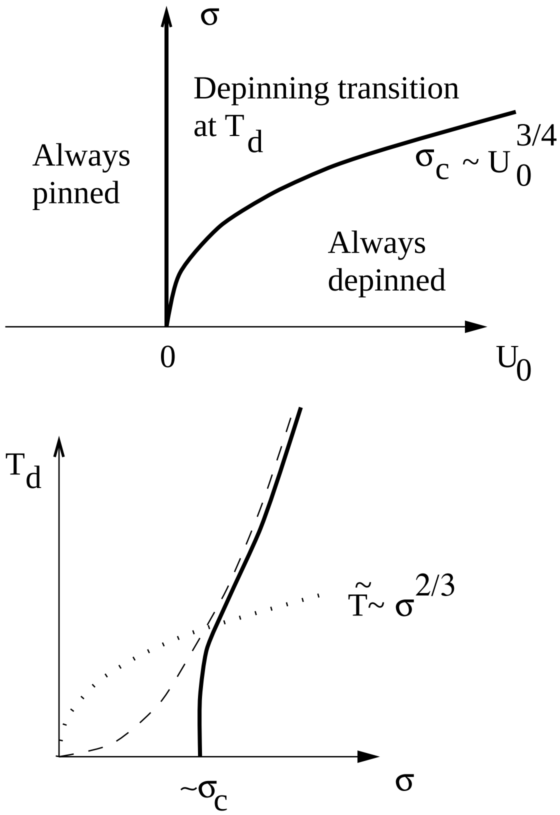

FIG. 3. *Top* : Diagram depicting three possible scenarios as a function of σ and U_0. *Bottom* : Depinning temperature T_b as a function of disorder, for a "repulsive" defect with $U_0 > 0$. The diagrammatic series gives a good approximation to the pinning free energy whenever $T \gg \tilde{T}$.

breaks down near a temperature $\tilde{T} \approx \left(\sigma_c^2/2\sigma^2\right)^{2/3} T_d$. For $T < \tilde{T}$, the localization length becomes of order b and energetic contributions dominate over entropic contributions to the free energy. At $T = 0$, the FL sits adjacent to the defect and makes excursions into it whenever energy fluctuations are favorable. The energy (per unit length) cost of a typical excursion of length ℓ is

$$E(\ell)/\ell \approx \tilde{\epsilon} b^2/\ell^2 + U_0 - \sqrt{\sigma^2 \xi/\ell}, \tag{28}$$

which is minimized for $\ell^* \approx (16\tilde{\epsilon}^2 b^4/\sigma^2\xi)^{1/3}$, and yields an estimate for the ground state energy

$$f(T=0) \approx -\frac{U_0}{2}\left[\frac{3}{4}\left(\frac{2\sigma^2}{\sigma_c^2}\right)^{2/3} - 1\right]. \tag{29}$$

For $\sigma \lesssim \sigma_c$, fluctuations are not favorable enough to make excursions into the defect, and the FL is depinned at all temperatures, which suggests why the perturbation expansion breaks down for this case.

The $T = 0$ behavior can be expected persist to finite temperatures as long as $\ell^* < \xi_z$, or equivalently, $T \gtrsim \tilde{T}$. Thus, as temperature is increased, there is a simple crossover from strong to weak pinning near $\tilde{T}$. Fig. 3 depicts a cartoon phase diagram and the weak pinning regime where the perturbation expansion is useful.

Finally, the same methodology can be simply generalized to a $(d+1)$-dimensional geometry. The perturbation series diverges for $d = 1$ as amply demonstrated by various authors [4–6], but useful results for all $d > 1$ can be obtained. In particular, results for the periodic potential [9] can be readily extended to the random case.

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[1] G. Blatter *et al.*, Rev. Mod. Phys. **66**, 1125 (1994).
[2] L. Civale *et al.*, Phys. Rev. L **67**, 648 (1991).
[3] D. R. Nelson and V. M. Vinokur, Phys. Rev. B **48**, 13 060 (1993).
[4] G. Forgacs, J. M. Luck, Th. M. Nieuwenhuizen, and H. Orland, Phys. Rev. L **57**, 2184 (1986); J. Stat. Phys. **51**, 29 (1988).
[5] B. Derrida, V. Hakim, and J. Vannimenus, J. Stat. Phys. **66**, 1189 (1992).
[6] H. Kallabis and M. Lässig, Phys. Rev. L **75**, 1578 (1995).
[7] S. M. Bhattacharjee and S. Mukherji, Phys. Rev. L **70**, 49 (1993); S. Mukherji and M. Bhattacharjee, Phys. Rev. E **48**, 3483 (1993).
[8] S. Nechaev and Y.-C. Zhang, Phys. Rev. L **74**, 1815 (1995)
[9] S. Galluccio and R. Graber, Phys. Rev. E **53**,R5584 (1996).
[10] M. Kardar, Nucl. Phys. **B290 [FS20]**, 582 (1987).